Time-variant reliability analysis of cooling towers including corrosion of steel in reinforced concrete

Analyse fiabiliste du vieillissement des aéro-réfrigérants dû à la corrosion des armatures

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Summary
Natural-draught cooling towers are used in nuclear power plants as heat exchangers. These structures are submitted to environmental loads such as wind and thermal gradients that are stochastic in nature. A probabilistic framework has been developed by EDF (Electricité de France) for assessing the durability of such structures. In this paper, the corrosion of the rebars due to concrete carbonation and the corresponding weakening of the reinforced concrete sections is considered. Due to the presence of time in the definition of the limit state function associated with the loss of serviceability of the cooling tower, time-variant reliability analysis has to be used. A novel approach is proposed to take into account the random “initiation time”, which corresponds to the time necessary for the carbonation to attain the rebars. Results are given in terms of the probability of failure of the structure over its life time.

Résumé
Les aéro-réfrigérants sont utilisés pour le refroidissement du circuit secondaire des centrales nucléaires. Ces structures sont soumises en fonctionnement à différents chargements qui sont par nature aléatoires (vent, gradient thermique, etc). Un cadre d’analyse probabiliste a été développé par EDF (Electricité de France) pour justifier leur tenue dans le temps. Dans cette communication, on considère comme unique phénomène de dégradation la corrosion des armatures dans le béton armé due à la carbonatation du béton, et l’affaiblissement progressif des sections qui en résulte. Le paramètre temporel intervient explicitement dans la définition de la fonction d’état limite choisie. Il est donc nécessaire de se placer dans le contexte d’une analyse de fiabilité dépendant du temps (time-variant). Une approche originale est proposée pour prendre en compte l’aléa dans la phase d’initiation de la corrosion, qui correspond au temps nécessaire pour que la carbonatation du béton atteigne les armatures. Les résultats obtenus sont la probabilité de défaillance de la structure en fonction du temps tout au long de la durée de vie.

Keywords : time-variant reliability, cooling towers, reinforced concrete, corrosion, finite element reliability.

Introduction
Natural-draught cooling towers are used in nuclear power plants as heat exchangers. These shell structures are submitted to environmental loads such as wind and thermal gradients that are stochastic in nature. Due to the complexity of the building procedure, uncertainties in the material properties as well as differences between the theoretical and the real geometry also exist.

In order to better understand the behaviour of these structures and develop a maintenance policy, Electricité de France (EDF) has developed tools for the reliability analysis of cooling towers in the past few years [1],[2]. Based on a probabilistic description of the input data such as material properties and loads and a limit state function describing the loss of serviceability of the cooling tower, a reliability index can be computed. Its evolution in time helps decision-making regarding maintenance.

In this paper, influence of the corrosion of the rebars in reinforced concrete on the reliability of the cooling tower is investigated. This leads to casting the problem as a time-variant reliability analysis.

The deterministic model including the modelling of corrosion is first discussed. Then the framework of structural reliability (in time-invariant and in time-variant context) is recalled. The deterministic assessment criterion and its probabilistic counterpart are then presented. The novel procedure developed in order to treat properly the random kinetics of the degradation phenomenon is then discussed. Numerical results showing the loss of reliability of the structure in time are finally given.

Deterministic model

Geometry and finite element model
The cooling tower under consideration is a shell structure with axisymmetric geometry depicted in Figure 1. The height of the tower is 165 m, the meridian line is described by Eq.(1) yielding the shell radius $R_m(z)$ as a function of the altitude $z$.

$$R_m(z) = -0.00263z + 0.2986\sqrt{(z - 126.74)^2 + 2694.5} + 23.333 \quad [1]$$

The shell is supported by 52 pairs of V-shaped columns with radius 0.5 m. The shell thickness is varying with $z$ from 21 cm to 120 cm.

The structure is modelled by using the finite element code ANSYS©. The mesh comprises 6344 4-node shell elements. The reinforced concrete is modelled as an homogeneous isotropic elastic material. The material properties, which are random variables in the present study, are given in the sequel.

Loading
When the power plant is in service, the cooling tower is submitted to the following combination of loads :

- the self-weight, corresponding to a mass density $\rho = 2500 \text{ kg/m}^3$.

Figure 1 : Cooling tower under consideration

\[ \text{Figure 1 : Cooling tower under consideration} \]
the wind pressure $q(z, \theta)$, which is codified in French Standard NV65, and depends on both altitude $z$ and azimuth $\theta$:

$$q(z, \theta) = \chi \cdot 2.5 \cdot \frac{z + 18}{z + 60} \cdot g(\theta)$$  \[2\]

In this equation, $P_0 = 700$ Pa is the reference wind pressure, $\chi = 1.2$ is a coefficient that takes into account the site and shell roughness influence, and $g(\theta)$ is represented as a 10-term Fourier series expansion. Variations of the wind pressure with altitude and azimuth is given in Figures 2a, 2b.

- the internal depression $P_{int}$ due to air circulation in service, which is constant all over the tower and is equal to 0.4 $P_{max}$ ($P_{max}$ being the maximal wind pressure at the top of the cooling tower), that is $P_{int} = -569$ Pa.
- the thermal gradient within the shell thickness. It is computed from the difference between the inside- and outside air temperatures and from the air/concrete heat transfer coefficient. In this study, the value $\nabla T = T_{int} - T_{ext} = 12 \degree$C is taken into account.
- differential settlements of the raft on which the cooling tower is founded. Vertical displacement obtained from in-situ measurement are introduced in the finite element model as “imposed displacements” to the base of the supporting columns.

**Corrosion model**

In this paper, corrosion of the rebars in reinforced concrete due to concrete carbonation is taken into account. This phenomenon can be summarized as follows: due to the porosity, CO$_2$ can penetrate the concrete and chemically react with the cement. This reaction involves OH$^-$ ions, which leads to a pH reduction. As soon as the low pH carbonated layer has attained a thickness, corrosion starts.

$$c(t) = \frac{1}{2} k f(h) - \sqrt{t}$$  \[3\]

where the carbonation depth increases with time as follows [3]:

$$x_c(t) = K_{carb} \sqrt{t} (\text{year}) \quad (\text{cm})$$  \[3\]

The corresponding steel cross-section used in the reinforced concrete assessment is obtained as:

$$A_e(t) = n \frac{\pi}{4} d^2(t)$$  \[6\]

where $n$ is the number of bars in the concrete section under consideration.

**Deterministic assessment of the structure**

In the present study, the serviceability of the cooling tower as a reinforced concrete structure is assessed according to the French concrete design code BAEL [6]. The failure of the most stressed section is checked. Such a vertical section has been selected from the results of a deterministic finite element computation (Figure 3).

Practically speaking, the computed internal forces $(N_{int}, M_{int})$ are reported in a load carrying capacity diagram (bending moment vs. tensile force) (see Figure 4), which is the locus of the internal forces that can be taken by the concrete section. This diagram is computed according to BAEL [6], and depends on the following parameters: geometry of the concrete section, rebars diameter and density, concrete cover, concrete compressive strength and steel yield stress.

If the representative point lies inside the diagram, the design criterion is satisfied for the section under consideration (e.g. the most stressed in the present case).
These random variables appear implicitly in Eq.(7) as follows: consequence, six independent random variables are used. As a definition in Table 1. It is emphasized that two variables of failure of the structure due to uncertainty in the material properties, loads, geometry, etc. Thus it requires:

- the probabilistic modelling of the parameters involved in the deterministic model, that is the definition of random variables (probability density function and associated parameters),
- the definition of a failure criterion by means of a limit state function

Time-invariant analysis is suited to quasi-static problems that involve only deterministic parameters and random variables. When stochastic processes or functions of time are explicitly present in the limit state function, time-variant analysis is required.

Reliability analysis

Reliability analysis aims at estimating the probability of failure of the structure due to uncertainty in the material properties, loads, geometry, etc. Thus it requires:

<table>
<thead>
<tr>
<th>Random variable</th>
<th>Type</th>
<th>Mean</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete compressive strength ( f_{c_f} )</td>
<td>lognormal</td>
<td>40 MPa</td>
<td>15 %</td>
</tr>
<tr>
<td>Concrete Young’s modulus ( E )</td>
<td>-</td>
<td>11 000 MPa</td>
<td></td>
</tr>
<tr>
<td>Concrete mass density ( p )</td>
<td>lognormal</td>
<td>2500 kg/m³</td>
<td>10 %</td>
</tr>
<tr>
<td>Concrete thermal dilatation coefficient ( \alpha )</td>
<td>lognormal</td>
<td>10⁻⁷ K</td>
<td>10 %</td>
</tr>
<tr>
<td>Steel yield stress ( f_y )</td>
<td>lognormal</td>
<td>420 MPa</td>
<td>5 %</td>
</tr>
<tr>
<td>Wind pressure ( P_c )</td>
<td>Gaussian</td>
<td>700 Pa</td>
<td>30 %</td>
</tr>
<tr>
<td>Internal depression ( P_{int} )</td>
<td>-</td>
<td>-0.813 ( P_c )</td>
<td></td>
</tr>
<tr>
<td>Exposure factor ( \gamma ) (carbonation depth)</td>
<td>lognormal</td>
<td>1.3</td>
<td>10 %</td>
</tr>
</tbody>
</table>

Random variables and distributions

The type and parameters of the random variables used in the analysis are defined in Table 1. It is emphasized that two variables are defined implicitly as functions of others, namely the Young’s modulus of concrete and the internal depression in service. As a consequence, six independent random variables are used.

These random variables appear implicitly in Eq.(7) as follows:

- \( E, \rho, P_c, P_{int} \) appear in the finite element computation, hence in \((N_{\text{ov}}, M_{\text{ov}})\), but also in \((N_{\text{ul}}, M_{\text{ul}})\) due to the method used to evaluate the latter.
- \( f_{c_f}, f_y \) appear in the load carrying capacity diagram
- \( \gamma \) appears in the initiation time (Eq.(14)).

It has been observed that the concrete cover has little importance (as a random variable) and can thus be taken as deterministic.

Degradation phenomena such as corrosion of rebars pertain to the latter class of problems, since the rebars diameter is explicitly defined as a function of time in Eq.(5).

Limit state function

The loss of serviceability is defined as the failure of the most stressed concrete section as in the deterministic case. The corresponding limit state function should be defined in such a way that positive values correspond to the “safe state” and negative values correspond to the “failure state”. Thus the following expression is adopted (Figure 5):

\[
g = (h^2 N_{\text{ov}} + M_{\text{ov}}^2) - (h^2 N_{\text{ul}} + M_{\text{ul}}^2)
\]

where \((h N_{\text{ov}}, M_{\text{ov}})\) are the internal forces resulting from the finite element computation (i.e. tensile force and bending moment related with a vertical section of the shell) and \((h N_{\text{ul}}, M_{\text{ul}})\) are obtained as the intersection between the radial line \((0,0)\) and the diagram boundary.

Time-invariant reliability analysis

If the limit state function does not depend on time, the probability of failure of the structure is computed as follows:

\[
P_f = P(g(X) \leq 0) \tag{8}
\]

where \(X\) is the vector of random variables. When a closed-form limit state function is available, techniques such as Monte Carlo or directional simulations are efficient. However these techniques require a large number of calls to the computation of the limit state function, especially when small probabilities of failure are sought for. Thus they are not applicable in practice to the current situation, where each evaluation of the limit state function requires a finite element run.

Alternatively, approximate methods such as FORM are attractive. The limit state function is recast in the standard normal space by using a probabilistic transformation \(X \leftrightarrow \tilde{U}\) (of Nataf or Rosenblatt type [7]). Then the problem is transformed into that of finding the design point \(\tilde{U}^*\), which minimizes the distance from the
The initiation time connected to analysis would reduce to a classical time-invariant analysis. The probability of failure of a structure in a given time range \([0, t]\) is given by the integral:

\[
P_f(t) = \Phi(-\beta_{\text{FORM}}(t))
\]

where the limit state function \(\Psi(t)\) is computed using a quadrature scheme.

A direct coupling approach between the finite element code ANSYS© and the reliability code RYFES is used [10, 11]. Each iteration of the optimization leading to the design point, a realization of the random variables is provided by RYFES to ANSYS. A finite element analysis is run with these values, from which the stresses and thus the limit state function is computed.

**Time-variant analysis : Practical treatment of the uncertainty on \(\gamma\)**

Time-variant reliability problems deal with computing the probability of failure of a structure in a given time range \([0, \tau]\):

\[
P_f(\tau) = P(\exists t \in [0, \tau], \, g(t, \mathbf{X}, \Psi(t)) \leq 0)
\]

where the arguments of the limit state function \(\Psi(t)\) depending on time can be stochastic processes, functions of time multiplied by random variables or both.

Problems involving stochastic processes are usually solved using the so-called "outcrossing approach" [12], where the mean number of outcrossing is evaluated by asymptotic formulae. However such an asymptotic approach is not applicable when the limit state function contains only functions of time and no random process, which is the case in the present study. Thus an alternative approach has to be followed.

In the present study, only the duration of the initiation phase (Eq.(4)) is supposed to be random due to uncertainties in the environmental loads. Randomness is characterized solely by the exposure factor \(\gamma\). Once the initiation phase is finished, the corrosion phenomenon makes the limit state function monotonically decreasing in time. This can be shown by noticing that a reduced rebars cross-section leads to a strictly smaller interaction diagram, and thus to a smaller "distance" between any point \((N_{\text{ref}}, M_{\text{ref}})\) and the boundary (in the sense of Eq.(7), see also Figure 5).

In this case, \(g(t) > g(\tau)\) for all \(t \in [0, \tau]\) whatever the realization of the random variables, Eq.(10) reduces to:

\[
P_f(t) = P(g(t) \leq 0)
\]

which means that the time-variant analysis reduces to a time-invariant one, where \(t\) is simply a parameter in the limit state function.

As a conclusion, if \(\gamma\) was a deterministic parameter, the whole analysis would reduce to a classical time-invariant analysis.

The initiation time connected to \(\gamma\) is actually random, due to the uncertainties in the environmental conditions. The dependency on \(\gamma\) is bypassed by using the conditional probability rule:

\[
P_f(t) = \int_0^\tau P_f(\tau | \gamma) f_\gamma(\tau) \, d\tau
\]

where \(f_\gamma(\tau)\) is the probability density function of \(\gamma\), which is lognormal in the present case. In this integral, the conditional probability reads:

\[
P_f(\tau | \gamma) = \begin{cases} P_{f,\text{init}} & \text{if } t \leq t_{\text{init}} \\ P_{f,\text{corrosion}}(t) & \text{if } t > t_{\text{init}} \end{cases}
\]

In the latter equation, \(P_{f,\text{init}}\) corresponds to the initial probability of failure computed with the initial steel cross-section, and \(P_{f,\text{corrosion}}(t)\) is the probability of failure at time \(t\) computed from Eq.(11) by a time-invariant analysis using the reduced steel cross-section defined in Eq.(6). It is emphasized that exposure factor \(\gamma\) which does not show in Eq.(13) is actually implicitly contained in \(t_{\text{init}}\) through Eq.(4).

The integration (12) is performed using a quadrature formula associated with the lognormal distribution \(f_\gamma(\tau)\) [13]:

\[
P_f(\tau) = \sum_{j=1}^{NP} \omega_j P_j(\gamma_j)
\]

where \(NP\) is the number of integration points. \((\gamma_j, \omega_j)\) are the integration-points and weights respectively. In the present study, 4 points were used, the values are given in Table 2.

<table>
<thead>
<tr>
<th>Point</th>
<th>Coordinate (\gamma_j)</th>
<th>Weight (\omega_j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0602276</td>
<td>0.0961941</td>
</tr>
<tr>
<td>2</td>
<td>1.2434957</td>
<td>0.5550763</td>
</tr>
<tr>
<td>3</td>
<td>1.4226822</td>
<td>0.3299973</td>
</tr>
<tr>
<td>4</td>
<td>1.9240603</td>
<td>0.0187323</td>
</tr>
</tbody>
</table>

**Numerical results**

Time-invariant analyses are performed for the values of \(\gamma\) reported in Table 2, and different points in time in the range \([0,60\text{ years}]\). For each time instant, the probability of failure is then computed by Eq.(14). The corresponding reliability index is computed by:

\[
\beta_{\text{FORM}}(t) = -\Phi^{-1}(P_f(\gamma))
\]

Results are reported in Figure 6. It is observed that the reliability index is almost constant during the first ten years. This corresponds to the initiation phase (penetration of carbonation), where the rebars cross-section is constant and equal to its original value. It is emphasized that the initiation time strongly depends on concrete cover. A (unlikely) small value was selected in the present analysis in order to illustrate the approach.

The reliability index later decreases as the corrosion evolves. Its evolution appears almost linear in time. In terms of maintenance, if a minimal reliability level was codified, the curve in figure 6 would give the time when repairing has to be started.

**Conclusion**

An novel approach for including properly degradation phenomena such as rebars corrosion in a time-variant reliability analysis has been proposed. The rebars corrosion associated with concrete carbonation presents an initiation phase, the duration of which is random due to the uncertainties in the environmental conditions. This randomness is accounted for by applying the conditional probability rule. Due to the strictly decreasing behaviour of the limit state function in time, the conditional time-variant probability of failure is computed by a single time-invariant analysis. The integration appearing in the conditional probability rule is computed using a quadrature scheme.

In the present paper, the kinetics of corrosion after its initiation is supposed deterministic. However, introducing randomness in its modelling would not add major difficulties. New random variables would simply be required in each time-invariant analysis necessary to compute \(P_f(\gamma_j)\) appearing in Eq.(14).

**References**


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Figure 6 : Evolution of the reliability index (resp. probability of failure) in time